UNIT – III

Measures of Central Tendency

(1) Find the Mean, Median and Mode from the following distribution:

Daily Sale in Rs.	No. of shops	x	fx
0 – 200	8	100	800
200 – 400	12	300	3600
400 – 600	30	500	15000
600 – 800	25	700	17500
800 – 1000	20	900	18000
1000 – 1200	5	1100	5500
	100		**60400**

Mean

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{60400}{100}$$

$$= 604$$

Solution:

Median:

Sale	No. of shops	cf
0 – 200	8	8
200 – 400	12	20
400 – 600	30	50
600 – 800	25	75
800 – 1000	20	95
1000 – 1200	5	100

$$\frac{N}{2} = 50$$

Med class 400 – 600

$$M = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{2} - cf\right)}{f}$$

$$= 400 + \frac{200(50 - 20)}{30}$$

$$= 400 + \frac{200 \times 30}{30}$$

$$= 600$$

Mode:

$$Z = l_1 + (l_2 - l_1) \frac{f_1 - f_0}{(2f_1 - f_0 - f_2)}$$

$$= 400 + \frac{200\ (18)}{60 - 12 - 25}$$

$$= 400 + \frac{200(18)}{23}$$

$$= 400 + 156.52 = 556.52$$

(2) Compute Median and Mode from the following distribution:

Rainfall in cms	No. of cities	cf
0 – 10	10	10
10 – 20	15	25
20 – 30	20	45
30 – 40	10	55
40 – 50	5	60

Solution:

Median:

$N = 60, \frac{N}{2} = 30$

Med class: Class containing $\frac{N}{2} = 30^{th}$ obs 20 – 30.

$$M = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{2} - cf\right)}{f}$$

$$= 20 + \frac{10(30 - 25)}{20}$$

$$= 20 + \frac{10(5)}{20} = 20 + 2.5 = 22.5$$

Mode:

$$Z = l_1 + (l_2 - l_1)\ \frac{f_1 - f_0}{(2f_1 - f_0 - f_2)}$$

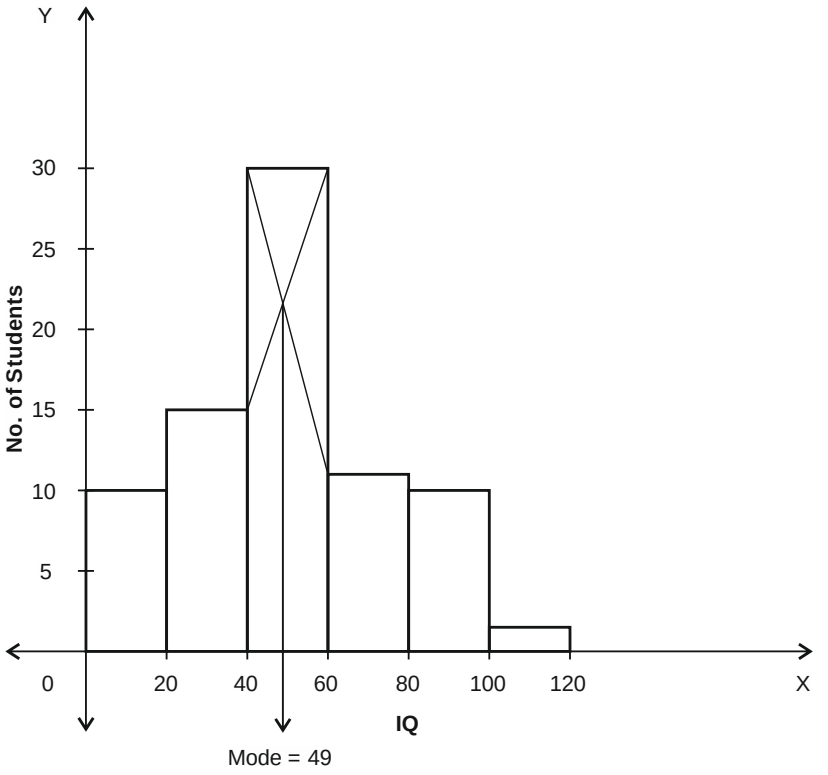
$$= 20 + \frac{10(20 - 15)}{2 \times 20 - 15 - 10}$$

$$= 20 + \frac{10(5)}{15} = 20 + 3.33 = 23.33$$

(3) Locate mode using Histogram for the following distribution:

IQ	0-20	20-40	40-60	60-80	80-100	100-120
No. of students	10	15	30	12	10	3

Solution:



Mode = 49

$$Z = l_1 + \frac{(l_2 - l_1)\,(f_1 - f_0)}{(2f_1 - f_0 - f_2)}$$

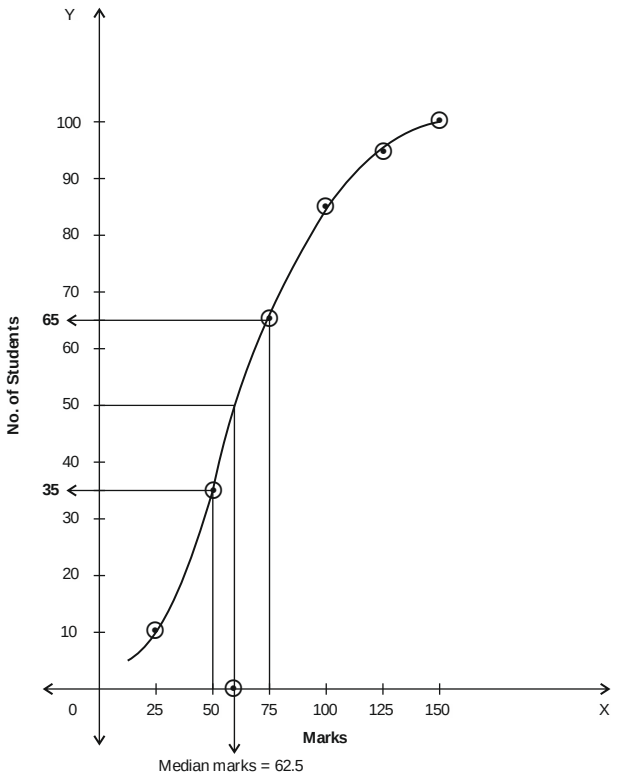
$$= 40 + \frac{20(15)}{33} = 40 + 9.09 = 49.09$$

(4) Draw less than Ogive for the following distribution:
Find (i) Median marks (ii) No. of students who have scored < 50.
(iii) No. of students who have scored > 75.

Marks	0-25	25-50	50-75	75-100	100-125	125-150
No. of students	10	25	30	20	10	5

Solution:

Marks	0-25	25-50	50-75	75-100	100-125	125-150
No. of students	10	25	30	20	10	5
cf	10	35	65	85	95	100



∴ No. of students who have scored < 50 = 35

∴ No. of students who have scored > 75 = 100 – 65 = 35.

(5) Compute 3rd Quartile, 7th Decile & 35th Percentile for the following distribution:

Commission in 000 Rs	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of Salesmen	7	20	25	18	15	10	5
cf	7	27	52	70	85	95	100

Solution:

3rd Quartile:

$$\frac{3N}{4} = \frac{3 \times 100}{4} = 75$$

$$Q_3 = l_1 + \frac{(l_2 - l_1)\left(\frac{3N}{4} - cf\right)}{f}$$

$$= 40 + \frac{10(75 - 70)}{15}$$

$$= 40 + \frac{10(5)}{15} = 43.33$$

7th Decile:

$$\frac{7N}{10} = \frac{7 \times 100}{10} = 70$$

$$D_7 = l_1 + \frac{(l_2 - l_1)\left(\frac{7N}{4} - cf\right)}{f}$$

$$= 30 + \frac{10(75 - 70)}{18}$$

$$= 40 + \frac{10(18)}{18} = 30 + 10 = 40$$

35th Percentile:

$$\frac{35N}{100} = 35$$

$$P_{35} = l_1 + \frac{(l_2 - l_1)\left(\frac{35N}{100} - cf\right)}{f}$$

$$= 20 + \frac{10(35 - 27)}{25}$$

$$= 20 + \frac{10(8)}{25} = 23.2$$

(6) Compute 1st Quartile, 4th Decile & 65th Percentile for the following distribution:

Production in tons	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of firms	10	15	20	25	15	10	5
cf	10	25	45	70	85	95	100

Solution:

1st Quartile: $\frac{N}{4} = 25$

$$Q_1 = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{4} - cf\right)}{f}$$

$$= 10 + \frac{10(25 - 10)}{15}$$

$$= 10 + \frac{10(15)}{15} = 10 + 10 = 20$$

4th Decile: $\frac{4N}{10} = 40$

$$D_4 = l_1 + \frac{(l_2 - l_1)\left(\frac{4N}{10} - cf\right)}{f}$$

$$= 20 + \frac{10(40 - 25)}{20}$$

$$= 20 + \frac{10(15)}{20} = 20 + 7.5 = 27.5$$

65th Percentile: $\frac{65N}{100} = 65$

$$P_{65} = l_1 + \frac{(l_2 - l_1)\left(\frac{65N}{100} - cf\right)}{f}$$

$$= 30 + \frac{10(65 - 45)}{25}$$

$$= 30 + \frac{10(20)}{25} = 30 + 8 = 38$$

(7) Compute three Quartiles for the following distribution:

Turnover in 000 Rs	0-10	10-20	20-30	30-40	40-50	50-60
No. of firms	20	35	45	30	20	10
cf	20	55	100	130	150	160

Solution:

Q_1 : $\frac{N}{4} = \frac{160}{4} = 40$

$$Q_1 = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{4} - cf\right)}{f}$$

$$= 10 + \frac{10(40 - 20)}{35}$$

$$= 10 + \frac{200}{15}$$

$$= 15.71$$

Q_2 : $\frac{N}{2} = \frac{160}{2} = 80$

$$Q_2 = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{2} - cf\right)}{f}$$

$$= 20 + \frac{10(80 - 55)}{45}$$

$$= 20 + \frac{10(25)}{45}$$

$$= 25.55$$

Q_3 : $\frac{3N}{4} = \frac{3 \times 160}{4} = 120$

$$Q_3 = l_1 + \frac{(l_2 - l_1)\left(\frac{3N}{4} - cf\right)}{f}$$

$$= 30 + \frac{10(120 - 100)}{30}$$

$$= 30 + \frac{10(20)}{30}$$

$$= 36.66$$

(8) Compute three Quartiles for the following distribution:

Marks Less than	20	30	40	50	60	70	80	90
No. of students	5	15	20	35	50	70	90	100

Solution:

Marks	0-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of students	= 5 5	= 15 – 5 10	= 20 – 15 5	= 35 – 20 15	= 50 – 35 15	= 70 – 50 20	= 90 – 70 20	= 100 –90 10
cf	5	15	20	35	50	70	90	100

Q_1 : $\frac{N}{4} = 25$

$$Q_1 = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{4} - cf\right)}{f}$$

$$= 40 + \frac{10(25 - 20)}{15}$$

$$= 40 + \frac{10(5)}{15} = 43.33$$

Q_2 : $\frac{N}{2} = 50$

$$Q_2 = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{2} - cf\right)}{f}$$

$$= 50 + \frac{10(50 - 35)}{15}$$

$$= 50 + \frac{10 \times 15}{15} = 60$$

Q_3 : $\frac{3N}{4} = 75$

$$Q_3 = l_1 + \frac{(l_2 - l_1)\left(\frac{3N}{4} - cf\right)}{f}$$

$$= 70 + \frac{10(75 - 70)}{20}$$

$$= 70 + \frac{50}{20} = 72.5$$

(9) The scores of 5 candidates in a selection procedure is given as follows :

Weights	3	4	1	2
Candidate	**Graduation**	**Written Test**	**Group Discussion**	**Personal Interview**
Ajay	60	73	45	51
Bipin	54	65	58	53
Kamalesh	62	80	60	52
Hetal	73	73	52	63
Jinal	75	90	62	55

Who will be the best candidate?

Solution:

Candidate	$\sum wx$	$\sum w$	$\bar{x} = \frac{\sum wx}{\sum w}$
Ajay	619	10	61.9
Bipin	586	10	58.6
Kamalesh	670	10	67
Hetal	689	10	68.9
Jinal	757	10	75.7

Jinal is the best candidate.

(10) The mean marks of a class of 100 students were 70. The mean marks of 45 boys was 50. Find mean marks of remaining girl students.

Solution:

Boys	**Girls**
$n_1 = 45$	$n_2 = 55$
$\bar{x}_1 = 50$	$\bar{x}_2 = ?$

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$70 = \frac{45 \times 50 + 55\,\bar{x}_2}{100}$$

$$7000 = 2250 + 55\,\bar{x}_2$$

$$\therefore \bar{x}_2 = 4750$$

$$\bar{x}_2 = 86.36$$

(11) The average production of a firm is 700 units. The average production by the morning shift employees is 500 units & that of evening shift is 800 units. Find the ratio of number of morning to number of evening shift employees.

Solution:

Morning	**Evening**	
$\bar{x}_1 = 500$	$\bar{x}_2 = 800$	$\bar{x} = 1000$

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$1700 = \frac{n_1 \times 500 + n_2 \times 800}{n_1 + n_2}$$

$$700\,n_1 + 700\,n_2 = 500\,n_1 + 800\,n_2$$

$$(700 - 500)n_2 = (800 - 700)\,n_2$$

$$200\,n_1 = 100\,n_2$$

$$\frac{n_1}{n_2} = \frac{100}{200}$$

$$n_1 : n_2 \quad 1:2$$

Measures of Dispersion

(1) Calculate Quartile Deviation for the following distribution:

Marks	No. of students	cf
0 – 10	5	5
10 – 20	15	20
20 – 30	20	40
30 – 40	10	50
40 – 50	10	60

Solution:

$$N = 60 \quad \frac{N}{4} = 15$$

Q_1 class: 10 – 20

$$Q_1 = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{4} - cf\right)}{f}$$

$$= 10 + \frac{10(15 - 5)}{15}$$

$$= 10 + \frac{100}{15} = 19.33$$

$$\frac{3N}{4} = 45$$

Q_3 class: 30 – 40

$$Q_3 = l_1 + \frac{(l_2 - l_1)\left(\frac{3N}{4} - cf\right)}{f}$$

$$Q_3 = 30 + \frac{10(45 - 40)}{10}$$

$$= 30 + 5 = 35$$

$$Q.D. = \frac{Q_3 - Q_1}{2}$$

$$= \frac{35 - 19.33}{2} = 7.833$$

(2) Calculate Quartile Deviation for the following distribution:

Weight	No. of Children	cf
0 – 20	3	3
20 – 40	10	13
40 – 60	15	28
60 – 80	12	40
80 – 100	8	48
100 – 120	12	60

Solution:

$$\frac{N}{4} = 15$$

Q_1 class: 40 – 60

$$Q_1 = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{4} - cf\right)}{f}$$

$$= 40 + \frac{20(15 - 13)}{15}$$

$$= 40 + \frac{20 \times 2}{15} = 45.86$$

$$\frac{3N}{4} = 45$$

Q_3 class: 80 – 100

$$Q_3 = l_1 + \frac{(l_2 - l_1)\left(\frac{3N}{4} - cf\right)}{f}$$

$$Q_3 = 80 + \frac{20(45 - 40)}{8}$$

$$= 80 + 12.5 = 92.5$$

$$Q.D. = \frac{Q_3 - Q_1}{2}$$

$$= \frac{92.5 - 45.86}{2} = 23.32$$

(3) Calculate Mean Deviation from Mean for the following distribution:

Age	**No. of policy holders**	**x**	**fx**	**|x – 37.5|**	**f|x – 37.5|**
10 – 20	5	15	75	22.5	112.5
20 – 30	10	25	250	12.5	125
30 – 40	20	35	700	2.5	50
40 – 50	15	45	675	7.5	112.5
50 – 60	10	55	550	17.5	175
	60		**2250**		**575**

Solution:

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{2250}{60} = 37.5$$

$$\text{M.D. from } \bar{x} = \frac{\sum f|x - \bar{x}|}{\sum f}$$

$$= \frac{575}{60}$$

$$= 9.58$$

(4) Calculate Mean Deviation from Median for the following distribution:

Monthly Rent in 000 Rs	**No of Families**	**cf**	**x**	**|x – 33|**	**f|x – 33|**
10 – 20	10	10	15	18	180
20 – 30	26	36	25	8	208
30 – 40	30	66	35	2	60
40 – 50	13	79	45	12	156
50 – 60	7	86	55	22	154
60 – 70	4	90	65	32	128
					886

Solution:

$$\frac{N}{2} = 45$$

Med class: 30 – 40

$$M = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{2} - cf\right)}{f}$$

$$= 30 + \frac{10(45 - 36)}{30}$$

$$= 30 + 3 = 33$$

$$\text{M.D. from median} = \frac{\sum f|x - M|}{\sum f}$$

$$= \frac{886}{90}$$

$$= 9.84$$

(5) Calculate Mean Deviation from Mode for the following distribution:

Electricity Consumption 000 units	**No. of firms**		**x**	**|x – 36|**	**f|x – 36|**
10 – 20	3		15	21	63
20 – 30	12	f_0	25	11	132
30 – 40	15	f_1	35	1	15
40 – 50	13	f_2	45	9	117
50 – 60	2		55	19	38
					365

Solution:

Modal class = 30 – 40

$$\text{Mode} = l_1 + \frac{(l_2 - l_1)\,(f_1 - f_0)}{2f_1 - f_0 - f_2}$$

$$= 30 + \frac{10(15 - 12)}{2 \times 15 - 12 - 13}$$

$$= 30 + \frac{10 \times 3}{5} = 36$$

$$\text{M.D. from median} = \frac{\sum f|x - \text{Mode}|}{\sum f}$$

$$= \frac{365}{45}$$

$$= 8.11$$

(6) Calculate Standard Deviation for the following distribution:

Profits 000 units	**No. of firms**	**x**	**fx**	**fx^2**
10 – 20	8	15	120	1800
20 – 30	17	25	425	10625
30 – 40	22	35	770	26950
40 – 50	33	45	1485	66825
50 – 60	15	55	825	45375
60 – 70	5	65	325	21125
			3950	**172700**

Solution:

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{3950}{100} = 39.5$$

$$\text{S.D.} = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$$

$$= \sqrt{\frac{172700}{100} - 39.5^2}$$

$$= \sqrt{1727 - 1560.25}$$

$$= \sqrt{166.75}$$

$$= 12.91$$

(7) Which of the following batsman is more consistent in his scores:

Runs scored by Batsman A	58	63	55	79	58	35	60	80
Runs scored by Batsman B	71	60	55	51	90	73	80	64

Solution:

$$\bar{x} = \frac{\sum x}{n} = \frac{488}{8} = 61 \qquad \bar{y} = \frac{\sum y}{n} = \frac{544}{8} = 68$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2} \qquad \sigma = \sqrt{\frac{\sum y^2}{n} - (\bar{y})^2}$$

$$= \sqrt{\frac{3118}{8} - 61^2} \qquad = \sqrt{\frac{38192}{8} - 68^2}$$

$$= \sqrt{3898.5 - 3721} \qquad = \sqrt{4774 - 4624}$$

$$= \sqrt{177.5} \qquad = \sqrt{150}$$

$$= 13.32 \qquad = 12.25$$

$$CV = \frac{\sigma}{\bar{x}} \times 100 \qquad CV = \frac{\sigma}{\bar{y}} \times 100$$

$$= \frac{13.32}{61} \times 100 \qquad = \frac{12.25}{68} \times 100$$

$$= 21.84 \qquad = 18.01$$

CV for batsman B is less.

$\therefore$ Batsman B is more consistent.

(8) Which of the following investment plan is more consistent in its returns:

Returns by Plan A	35	41	62	51	40	25	30	44
Returns by Plan B	46	65	49	64	58	71	50	61

Solution:

$$\bar{x} = \frac{\sum x}{n} = \frac{328}{8} = 41 \qquad \bar{y} = \frac{\sum y}{n} = \frac{464}{8} = 58$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2} \qquad \sigma = \sqrt{\frac{\sum y^2}{n} - (\bar{y})^2}$$

$$= \sqrt{\frac{14412}{8} - 41^2} \qquad = \sqrt{\frac{27464}{8} - 58^2}$$

$$= \sqrt{1801.5 - 1681} \qquad = \sqrt{3433 - 3364}$$

$$= \sqrt{120.5} \qquad = \sqrt{169}$$

$$= 10.98 \qquad = 8.3$$

$$CV = \frac{\sigma}{\bar{x}} \times 100 \qquad CV = \frac{\sigma}{\bar{y}} \times 100$$

$$= \frac{10.98}{41} \times 100 \qquad = \frac{8.3}{58} \times 100$$

$$= 26.77 \qquad = 14.32$$

CV for plan B is less.

$\therefore$ Plan B is more consistent.

(9) Calculate the combined Standard deviation for the following:

	Boys	**Girls**
Number	30	70
Mean Height	120	100
S.D. Height	9	5

Solution:

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{30 \times 120 + 70 \times 100}{30 + 70}$$

$$= \frac{10600}{100} = 106$$

$$d_1 = \bar{x} - \bar{x}_1$$

$$= 120 - 106$$

$$= 14$$

$$d_2 = \bar{x} - \bar{x}_2$$

$$= 100 - 106$$

$$= -6$$

$$\sigma = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

$$= \sqrt{\frac{30(9^2 + 14^2) + 70(5^2 + 6^2)}{100}}$$

$$= \sqrt{\frac{30(81 + 196) + 70(25 + 36)}{100}}$$

$$= \sqrt{\frac{8310 + 4270}{100}} = \sqrt{\frac{12580}{100}} = 11.22$$

(10) Calculate the unknown values:

	Men	**Women**	**Total**
Number	50	100	–
Mean Weight	70	–	60
Variance. Weight	9	–	225

Solution:

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$60 = \frac{50 \times 70 + 100 \times \bar{x}_2}{150}$$

$$9000 = 3500 + 100\,\bar{x}_2$$

$$100\,\bar{x}_2 = 5500$$

$$\bar{x}_2 = 55$$

$$d_1 = \bar{x} - \bar{x}_1$$
$$= 60 - 70$$
$$= -10$$

$$d_2 = \bar{x} - \bar{x}_2$$
$$= 60 - 55$$
$$= 5$$

$$\sigma = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

$$225 = \frac{50(9 + 100) + 100(\sigma_2^2 + 25)}{150}$$

$$33750 = 5450 + 100\,\sigma_2^2 + 2500$$

$$25800 = 100\,\sigma_2^2$$

$$258 = \sigma_2^2$$

$$\sigma_2 = 16.06$$

Unit IV

Elementary Probability Theory

(1) A box contains 5 red, 3 Green & 2 black pens. 3 pens are drawn from the box. What is the probability that (i) They are of same colour (ii) They are of different colours.

Solution:

$\boxed{5R\ \ 3G\ \ 2B} \rightarrow 3$

$n(S) = {}^{10}C_3$

(i) P(3 pens are of same colour) = P(3 Red) + P(3 Green) + (3 black)

Working $= \frac{{}^{5}C_3}{{}^{10}C_3} + \frac{{}^{3}C_3}{{}^{10}C_3} + 0 = \frac{10}{120} + \frac{1}{120} + 0$

$= \frac{10 + 1}{120} = \frac{11}{120}$

(ii) P(they are of diff colour) = 1 – P (same colour)

$= 1 - \frac{11}{120} = \frac{109}{120}$

(2) There are 2 managers, 5 officers & 3 clerks in a department. A committee of 3 is to be formed. What is the probability that the committee contains:
(i) No clerk (ii) At least one clerk (iii) At least 2 officers

Solution:

$n(S) = {}^{10}C_3 = 120$

(i) P(No clerk) $= \frac{{}^{7}C_3}{{}^{10}C_3} = \frac{35}{120}$

(ii) P(At least one clerk) $= 1 - P(\text{No clerk}) = 1 - \frac{35}{120} = \frac{85}{120}$

(iii) P(At least 2 officers) = P(2 officers) + P(3 officers)

$= \frac{{}^{5}C_2 \times {}^{5}C_1 + {}^{5}C_3}{{}^{10}C_3}$

$= \frac{10 \times 5 + 10}{120} = \frac{60}{120} = \frac{1}{2}$

(3) Two fair dice are rolled What is the probability that the sum of the numbers on uppermost faces (i) Less than 7 (ii) Multiple of 3 (iii) Perfect square.

Solution:

$$n(S) = 36$$

A: Sum of numbers < 7

$$P(A) = \{(1,1), (1,2), (2,1), (1,3), (2,2), (3,1), (1,4), (2,3), (3,2), (4,1), (1,5), (2, 4), (3,3), (4,2), (5,1)\}$$

$$n(A) = 15$$

$$P(A) = \frac{15}{36} = \frac{5}{12}$$

B: Multiple of 3 is 3, 6, 9, 12.

B: {(1,2), (2,1), (1,5), (2,4), (3,3), (4,2), (5,1), (3,6), (4,5), (15,4), (6,3), (6,6)}

$$n(B) = 12$$

$$P(B) = \frac{12}{36} = \frac{1}{3}$$

C: Perfect square 4 and 9.

$$P(C) = \{(1,3), (2,2), (3,1), (3,6), (4,5), (5,4), (6,3)\}$$

$$n(C) = 7$$

$$P(C) = \frac{7}{12}$$

(4) Three unbiased coins are tossed. What is the probability that the tosses show:
(i) Two heads (ii) No head (iii) At least two tails.

Solution:

$$S = \{HHH, HTT, THT, TTH, THH, HTH, HHT, TTT\}$$

A: 2 heads

$$A = \{THH, HTH, HHT\}$$

$$P(A) = \frac{3}{8}$$

B: No head

$$B = \{TTT\}$$

$$P(B) = \frac{1}{8}$$

C: At least two tails.

$$C = \{HTT, THT, TTH, TTT\}$$

$$n(C) = 4$$

$$P(C) = \frac{4}{8} = \frac{1}{2}$$

(5) Eight students including a pair of twins are seated for a photograph. What is the probability that (i) The twins are together (ii) The twins are at the extremes?

Solution:

$$P(\text{Twins are together}) = \frac{7!\,2!}{8!} = \frac{2}{8} = \frac{1}{4}$$

$$P(\text{Twins are at the extremes}) = \frac{2 \times 6!}{8!} = \frac{1}{28}$$

(6) The time table for an examination is to be framed. There are seven subjects including a two papers in Management. What is the probability that (i) The Management papers are together (ii) Management papers are on 2^{nd} and 5^{th} day?

Solution:

$$P(\text{Management papers together}) = \frac{6!\,2!}{8!} = \frac{1}{28}$$

$$P(\text{Mgmt papers are on } 2^{nd} \text{ and } 5^{th} \text{ day}) = \frac{1 \times 5!}{8!} = \frac{1}{336}$$

(7) Two cards are drawn from a well shuffled pack of cards. What is the probability that (i) Both are red cards (ii) Both are picture cards (ii) One is an Queen and other is King (iv) One is Club and other is Heart card?

Solution:

$$n(S) = {}^{52}C_2$$

A: Both are red

$$P(A) = \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{25}{102}$$

B: Both are picture cards.

There are $3 \times 4 = 12$ picture cards.

$$P(B) = \frac{{}^{12}C_2}{{}^{52}C_2} = \frac{12 \times 11}{52 \times 51} = \frac{11}{221}$$

C: One is Queen and other is King.

$$P(C) = \frac{{}^{4}C_1 \times {}^{4}C_1}{{}^{52}C_2} = \frac{8}{663}$$

D: One is club and other is hear card.

There are 13 club cards and 13 heart cards.

$$P(D) = \frac{{}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_2} = \frac{13}{102}$$

(8) The odds in favour of Seema wining the contest is 2 : 3. The odds against Reema wining the contest is 3:4 What is the probability that:
(i) Seema wins the contest (ii) Reema wins the contest (iii) Exactly one of them wins the contest.

Solution:

A: Seema wins.

$$P(A) = \frac{2}{5} \qquad P(\bar{A}) = \frac{3}{5}$$

B: Reema wins.

$$P(B) = \frac{4}{7} \qquad P(\bar{B}) = \frac{3}{7}$$

$$P(\text{Seema wins}) = P(A \cap B) + P(A \cap \bar{B})$$

$$= P(A) \times P(B) + P(A) \times P(\bar{B})$$

$$= \frac{2}{5} \times \frac{4}{7} + \frac{2}{5} \times \frac{3}{7} = \frac{2}{5}$$

$$P(\text{Reema wins}) = P(A \cap B) + P(\bar{A} \cap B)$$

$$= P(A) \times P(B) + P(\bar{A}) \times P(B)$$

$$= \frac{2}{5} \times \frac{4}{7} + \frac{3}{5} \times \frac{4}{7} = \frac{4}{7}$$

$$P(\text{Exactly one wins}) = P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$$

$$= \frac{2}{5} \times \frac{3}{7} + \frac{3}{5} \times \frac{4}{7}$$

$$= \frac{6 + 12}{35} = \frac{18}{35}$$

(9) For the following distribution calculate:
(i) P(X > 0) (ii) P(X <= 1) (iii) E(X) (iv) V(X)

X	**P(X)**	**x.P(X)**	**x^2 P(X)**
– 2	0.05	– 0.1	0.2
– 1	0.15	– 0.15	0.15
0	k	0	0
1	0.3	0.3	0.3
2	0.2	0.4	0.8
3	0.1	0.3	0.9
		0.75	**2.35**

$\sum P(x) = 1$

$0.8 + k = 1$

$k = 0.2$

Solution:

$P(X > 0) = P(x = 1) + (P(x = 2) + P(x = 3)$

$= 0.3 + 0.2 + 0.1 = 0.6$

$P(X <= 1) = P(X = -2) + 2(X = -1) + P(X = 0) + P(X = 1)$

$= 0.05 + 0.15 + 0.2 + 0.3$

$= 0.7$

$E(x) = \sum x.P(x) = 0.75$

$E(x^2) = \sum x^2 P(x) = 2.35$

$V(x) = E(x^2) - [E(x)]^2$

$= 2.35 - (0.75)^2$

$= 2.35 - 0.5625 = 1.7875$

(10) If X is a random variable with probability mass function P(X = x) = kx; x = 0, 1, 2, 3, 4, 5 = 0 otherwise
Find (i) K (ii) E(X) (iii) V(X)

Solution:

X	**0**	**1**	**2**	**3**	**4**	
P(X)	kx	kx	kx	kx	kx	$\sum P(X) = 1$
	= 0	k	2k	3k	4k	$\therefore 10k = 1$
	= 0	0.1	0.2	0.3	0.4	$k = 0.1$
X.P(X)	0	0.1	0.4	0.9	1.6	

$E(x) = \sum x.P(x) = 3$

$E(x^2) = \sum x^2 P(x) = 0 + 0.1 + 0.8 + 2.7 + 6.4 = 10$

$V(x) = E(x^2) - (E(x))^2 = 10 - 3^2 = 10 - 9 = 1$

(11) In a certain business an entrepreneur can make a profit of Rs. 1,00,000 with probability 0.4 or suffer a loss of Rs. 50,000 with probability 0.6. Calculate expected profit of the entrepreneur.

Solution:

X	**Profit**	**Loss**
	1,00,000	– 50,000
P(X)	0.4	0.6

$E(x) = \sum X.P(x)$

$= 40,000 - 30,000$

$= 10,000$

UNIT V

Decision Theory

(1) A fruit seller has the option of buying 20, 40 or 60 watermelons at a rate of Rs. 50 per watermelon. He can sell each watermelon at the rate of Rs. 70 per watermelon. He expects a demand for 20, 40 or 60 watermelons. Note that he has to discard the unsold watermelons. Form a pay off table.

Solution:

Purchase / Sale (demand)	20	40	60
20	$20 \times 70 - 20 \times 50 = 400$	$20 \times 70 - 40 \times 50 = -600$	$20 \times 70 - 60 \times 50 = -1600$
40	$20 \times 70 - 20 \times 50 = 400$	$40 \times 70 - 40 \times 50 = 800$	$40 \times 70 - 60 \times 50 = -200$
60	$20 \times 70 - 20 \times 50 = 400$	$40 \times 70 - 40 \times 50 = 800$	$60 \times 70 - 60 \times 50 = 1200$

(2) The demand for a seasonal product is given below:

Demand during the season	40	50	60	70
probability	0.2	0.3	0.35	0.15

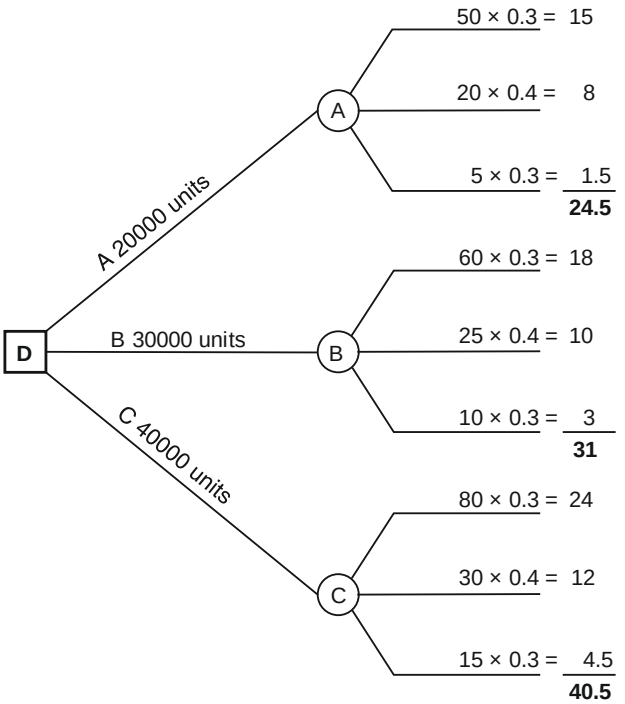
The product costs Rs. 60 per unit and is sold at Rs. 80 per unit. If units are not sold within the season, they will have no market value. Form a pay off table.

Solution:

		0.2	**0.3**	**0.35**	**0.15**
		40	**50**	**60**	**70**
	Purchase / Sale	**8**	**15**	**21**	**10.5**
8	40 × 0.2	80 × 8 – 60 × 8 = 160	80 × 8 – 60 × 15 = – 260	80 × 8 – 60 × 21 = – 620	80 × 8 – 60 × 10.5 = 10
15	50 × 0.3	80 × 8 – 60 × 8 = 160	80 × 15 – 60 × 15 = 300	80 × 15 – 60 × 21 = – 60	80 × 15 – 60 × 10.5 = 570
21	60 × 0.35	80 × 8 – 60 × 8 = 160	80 × 15 – 60 × 15 = 300	80 × 21 – 60 × 21 = 420	80 × 21 – 60 × 10.5 = 1050
10.5	70 × 0.15	80 × 8 – 60 × 8 = 160	80 × 15 – 60 × 15 = 300	80 × 21 – 60 × 21 = 420	80 × 10.5 – 60 × 10.5 = 210

(3) A auto company has to decide about the size of their new plant. Three alternatives of annual capacity (A) 20000 units (B) 30000 units and (C) 40000 units. The estimated profits for plant - A are 50 cr, 20 cr and 5 cr, if the demand is high, fair and low respectively. The corresponding sale figures for plant - B are 60 cr, 25 cr and 10 cr and that for plant – C are 80 cr, 30 cr and 15 cr respectively. The probabilities of the demand being high, fair and low are 0.3, 0.4 & 0.3 respectively. Suggest the optimal decision using decision tree.

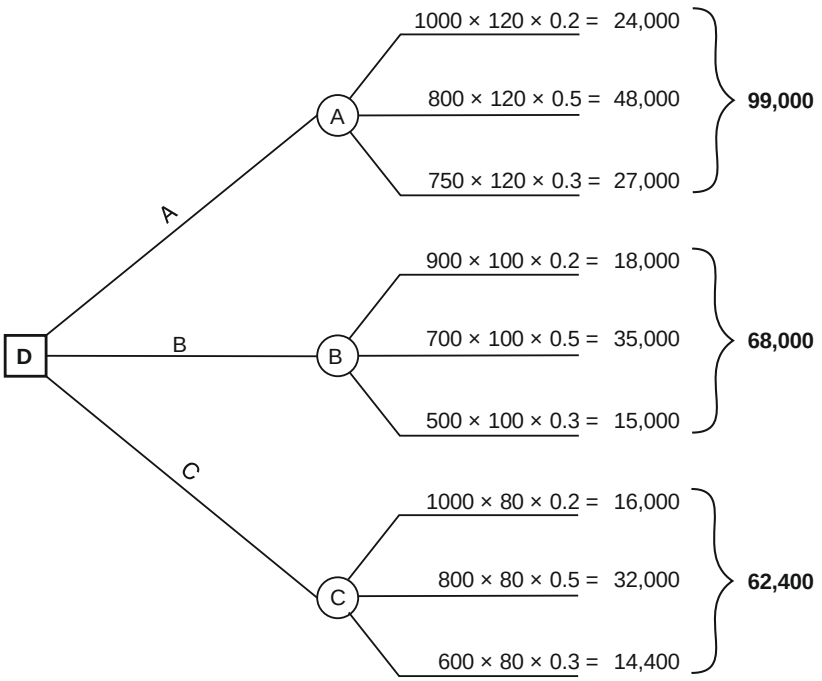
Solution:



Optimum decision is plan C i.e. 40,000 units.

(4) A company has to decide about the type of body lotions (A, B or C) to be produced. The estimated sales figures for type A body lotion are 1000 units, 800 units and 750 units if the demand is high, fair and low respectively. The corresponding sale figures type B body lotion are 900 units, 700 units and 500 units and that for type C body lotion are 1000 units, 800 units and 600 units respectively. The estimated profits per unit for the three types of body lotions are Rs. 120, Rs. 100 and Rs. 80 respectively. The probabilities of the demand being high, fair and low are 0.2, 0.5 and 0.3 respectively. Suggest the optimal decision using decision tree.

Solution:



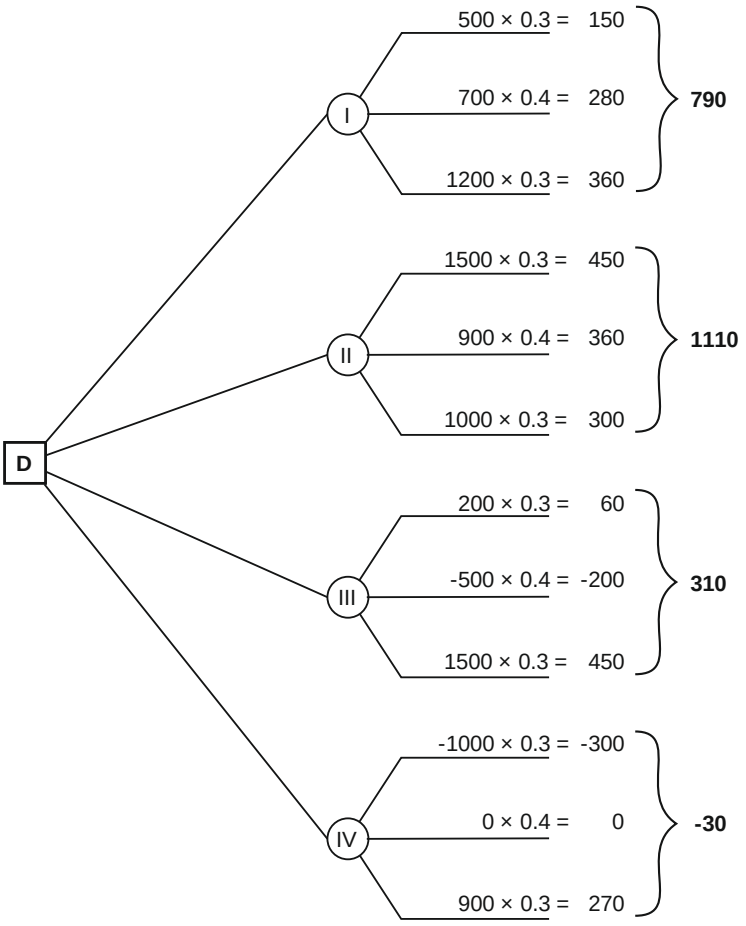
Optimum decision is type A body lotion.

(5) Associated Engineering company is evaluating four alternative investment options whose returns are based on the state of economy with following pay of matrix.

State of Economy	Profit in Rs.				Prob.
	Option 1	Option 2	Option 3	Option 4	
Fair	500	1500	200	– 1000	0.3
Good	700	900	– 500	0	0.4
Better	1200	1000	1500	900	0.3

Suggest the optimal decision using decision tree.

Solution:



Optimum decision is option II.

(6) The decision maker has 4 Courses of actions A1, A2, A3. A4 to choose from. There are 4 states of nature S1, S2, S3 & S4. Decide the best action using (i) EMV (ii) EOL methods.

	Course of Actions				
State of nature	**A1**	**A2**	**A3**	**A4**	**Prob.**
S1	20	30	35	40	0.2
S2	40	30	40	55	0.2
S3	25	40	50	65	0.3
S4	40	30	45	40	0.3

Solution:

$EMV(A_1) = 20 \times 0.2 + 40 \times 0.2 + 25 \times 0.3 + 40 \times 0.3 = 31.5$

$EMV(A_2) = 30 \times 0.2 + 30 \times 0.2 + 40 \times 0.3 + 30 \times 0.3 = 33$

$EMV(A_3) = 35 \times 0.2 + 40 \times 0.2 + 50 \times 0.3 + 45 \times 0.3 = 43.5$

$EMV(A_4) = 40 \times 0.2 + 55 \times 0.2 + 65 \times 0.3 + 40 \times 0.3 = 50.5$

Best Action is A_4.

Regret Table

	A1	**A2**	**A3**	**A4**	**Prob.**
S1	20	10	5	0	0.2
S2	15	25	15	0	0.2
S3	40	25	15	0	0.3
S4	5	15	0	5	0.3

$EOL(A_1) = 20 \times 0.2 + 15 \times 0.2 + 40 \times 0.3 + 5 \times 0.3 = 20.5$

$EOL(A_2) = 10 \times 0.2 + 25 \times 0.2 + 25 \times 0.3 + 15 \times 0.3 = 19$

$EOL(A_3) = 5 \times 0.2 + 15 \times 0.2 + 15 \times 0.3 + 0 \times 0.3 = 8.5$

$EOL(A_4) = 0 + 0 + 0 + 5 \times 0.3 = 1.5$

EOL is minimum for A_4.

$\therefore$ A_4 is the best.

(7) The decision maker has 4 Courses of actions A1, A2, A3, A4 to choose from. There are 4 states of nature S1, S2, S3 & S4. Decide the best action using (i) EMV (ii) EOL methods:

	Course of Actions				
State of nature	**A1**	**A2**	**A3**	**A4**	**Prob.**
S1	300	– 200	350	400	0.15
S2	500	700	– 200	500	0.35
S3	250	– 450	650	600	0.30
S4	– 450	500	600	– 700	0.20

Solution:

$EMV(A_1) = 300 \times 0.15 + 500 \times 0.35 + 250 \times 0.3 - 450 \times 0.2 = 205$

$EMV(A_2) = -200 \times 0.15 + 700 \times 0.35 - 450 \times 0.3 + 500 \times 0.2 = 180$

$EMV(A_3) = 350 \times 0.15 - 200 \times 0.35 + 650 \times 0.3 + 600 \times 0.2 = 297.5$

$EMV(A_4) = 400 \times 0.15 + 500 \times 0.35 + 600 \times 0.3 - 700 \times 0.2 = 275$

EMV is maximum for A_3.

$\therefore$ A_3 is the best.

Regret Table

	A1	**A2**	**A3**	**A4**	
S1	100	600	50	0	0.15
S2	200	0	900	200	0.35
S3	400	1100	0	50	0.3
S4	1050	100	0	1300	0.2

$EOL(A_1) = 100 \times 0.15 + 200 \times 0.35 + 400 \times 0.3 + 1050 \times 0.2 = 415$

$EOL(A_2) = 600 \times 0.15 + 0 \times 0.35 + 1100 \times 0.3 + 100 \times 0.2 = 620$

$EOL(A_3) = 50 \times 0.15 + 900 \times 0.35 + 0 + 0 = 322.5$

$EOL(A_4) = 0 + 200 \times 0.35 + 50 \times 0.3 + 1300 \times 0.2 = 345$

EOL is minimum for A_3.

$\therefore$ A_3 is the best.

(8) A company is to launch 3 models of motorbikes model I, model II & model III with estimated levels of demands Best, Better Good with probabilities 0.2, 0.5 & 0.3. Estimated profits in lacs of Rs. Is given below. Use (i) EMV (ii) EOL criteria to select the best decision.

	Course of Actions			
State of nature	**A1**	**A2**	**A3**	**Prob.**
Best	10	20	12	0.2
Better	8	8	8	0.5
Good	0	– 5	0	0.3

Solution:

$EMV(A_1) = 10 \times 0.2 + 8 \times 0.5 + 0 \times 0.3 = 6$

$EMV(A_2) = 20 \times 0.2 + 8 \times 0.5 - 5 \times 0.3 = 6.5$

$EMV(A_3) = 12 \times 0.2 + 8 \times 0.5 + 0 \times 0.3 = 6.4$

EMV is maximum for A_2.

$\therefore$ A_2 is the best.

Regret Table

	A1	**A2**	**A3**	**Prob.**
S1	10	0	8	0.2
S2	0	0	0	0.5
S3	0	5	0	0.3

$EOL(A_1) = 10 \times 0.2 = 2$

$EOL(A_2) = 5 \times 0.3 = 1.5$

$EOL(A_3) = 8 \times 0.2 = 4$

EOL is minimum for A_2.

$\therefore$ A_2 is the best.

(9) Given the following pay off table. Suggest the best action using (i) Maximin (ii) Maximax (iii) Minimax Regret (iv) Laplace criterion:

	Course of Actions			
State of nature	**A1**	**A2**	**A3**	**A4**
S1	300	900	– 35	0
S2	700	0	450	400
S3	– 400	100	0	800
S4	– 225	50	200	0

Solution:

Min	– 400	0	– 35	0
Max	700	900	450	800
Avg	93.75	262.5	153.75	300

Maximin = Max (– 400, 0, – 35, 0) = 0 ∴ best is A2, A4.

Maximax = Max (700, 900, 450, 800) = 900 ∴ best is A2.

Laplace = Max (93.75, 262.5, 153.75, 300) = 300 ∴ best is A4.

Regret Table

	A1	**A2**	**A3**	**A4**
S1	600	0	935	900
S2	0	700	250	300
S3	1200	700	800	0
S4	425	150	0	200
Max	1200	700	935	900

Minimax = Min (1200, 700, 935, 900)

= 700 ∴ best is A2.

(10) Given the following pay-off table suggest best course of action according to (i) Maximin, (ii) Maximax, (iii) Laplace and (iv) Minimax Regret Criterion:

	Course of Actions			
State of nature	**A1**	**A2**	**A3**	**A4**
S1	– 6000	– 4000	– 3000	– 2000
S2	1500	2000	0	1700
S3	1500	3000	2000	3500
S4	– 1000	0	3000	8000
S5	3000	4000	6000	10000

Solution:

Min	1500	– 4000	– 3000	– 2000
Max	3000	4000	6000	10000
Laplace	– 200	1000	1600	4240

Maximin = Max (1500, – 4000, – 3000, – 2000) = 1500 ∴ A1 is the best

Maximax = Max (3000, 4000, 6000, 10000) = 10000 ∴ A4 is the best

Laplace = Max (– 200, 1000, 1600, 4240) = 4240 ∴ A4 is the best

Regret Table

	A1	**A2**	**A3**	**A4**
S1	4000	2000	1000	0
S2	500	0	2000	300
S3	2000	500	1500	0
S4	9000	8000	5000	0
S5	7000	6000	4000	0
Max	9000	8000	5000	300

Minimax = Min (9000, 8000, 5000, 300)

= 300 ∴ A4 is the best

(11) Given the following pay off table. Suggest the best action using (i) Maximin (ii) Maximax (iii) Minimax Regret (iv) Laplace criterion:

	Course of Actions			
State of nature	**Plan A**	**Plan B**	**Plan C**	**Plan D**
High	– 200	600	0	– 300
Moderate	250	400	300	0
Low	450	– 800	500	150

Solution:

Min	– 200	– 800	0	– 300
Max	450	600	500	150
Laplace	166.6	66.6	266.6	– 50

Maximin = Max (– 200, – 800, 0, – 300) = 0 Plan C is the best.

Maximax = Max (450, 600, 500, 150) = 600 Plan B is the best.

Laplace = Max (166.6, 66.6, 266.6, – 50) = 266.6 Plan C is the best.

Regret Table

	A	**B**	**C**	**D**
S1	800	0	600	900
S2	150	0	100	400
S3	50	1300	0	350
Max	800	1300	600	900

Minimax Regret = Min (800, 1300, 600, 900)

= 600 Plan C is the best.